Network Analysis

B. E. Third Semester Electrical Engineering

Lecture: 03

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Complex Numbers

Complex Numbers were especially when dealing with frequency dependent sinusoidal sources and vectors to allow complex equations to be solved with numbers that are the square roots of negative numbers, $\sqrt{-1}$.

To represent imaginary number j-operator is used.
Eg. j3

Complex Numbers

Then a complex number consists of two distinct but very much related parts,
a “ Real Number ” plus an “ Imaginary Number ”.

Eg. 3+j4

Complex Numbers using the Rectangular Form

$$Z = x + jy$$

Where,

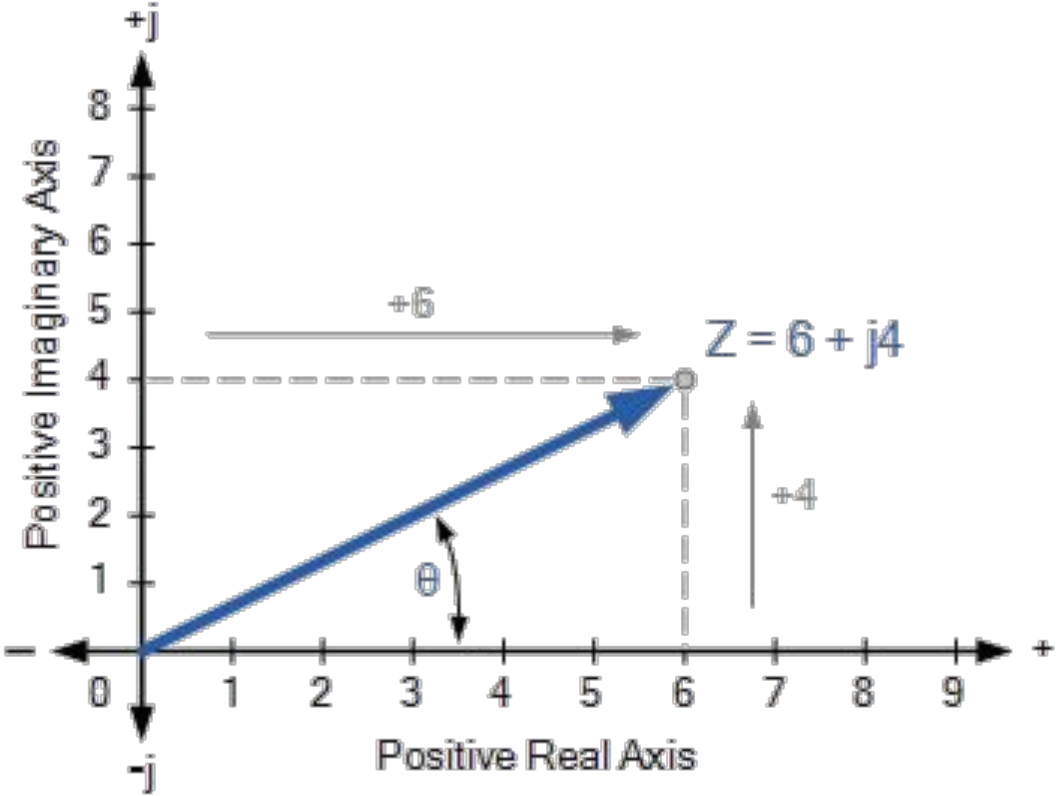
Z - Complex Number representing the Vector

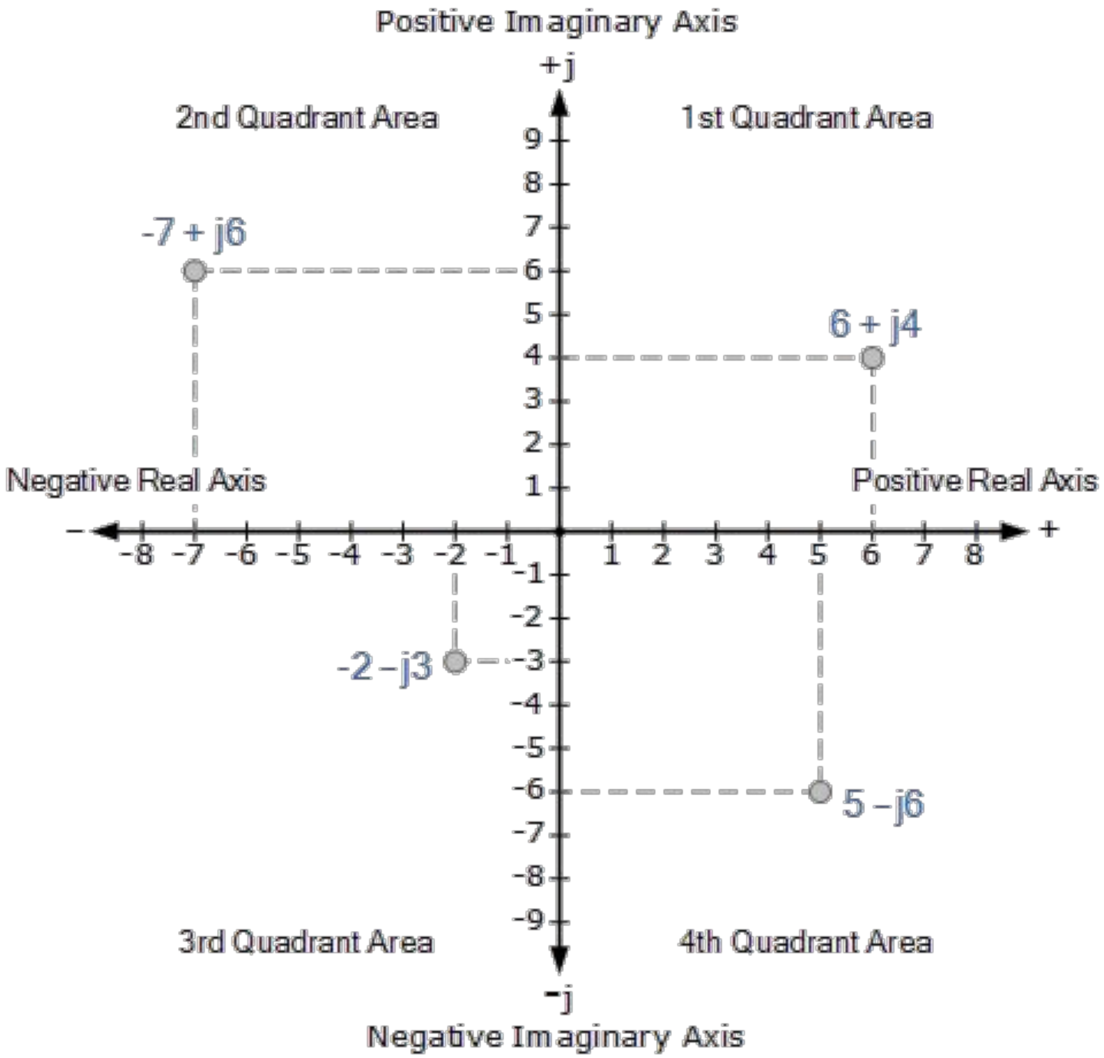
x - Real part or the Active component

y - Imaginary part or the Reactive component

j - is defined by $\sqrt{-1}$

Complex Numbers




Positive Imaginary Axis
+j
2nd Quadrant Area
1st Quadrant Area
-7 + j6
6 + j4
Negative Real Axis
Positive Real Axis
-2 – j3
5 – j6
3rd Quadrant Area
4th Quadrant Area
-j
Negative Imaginary Axis

Complex Addition and Subtraction

$$A = x + jy \qquad B = w + jz$$

$$A + B = (x + w) + j(y + z)$$

$$A - B = (x - w) + j(y - z)$$

Complex Addition and Subtraction

Addition

$$A + B = \left(4 + j1\right) + \left(2 + j3\right)$$

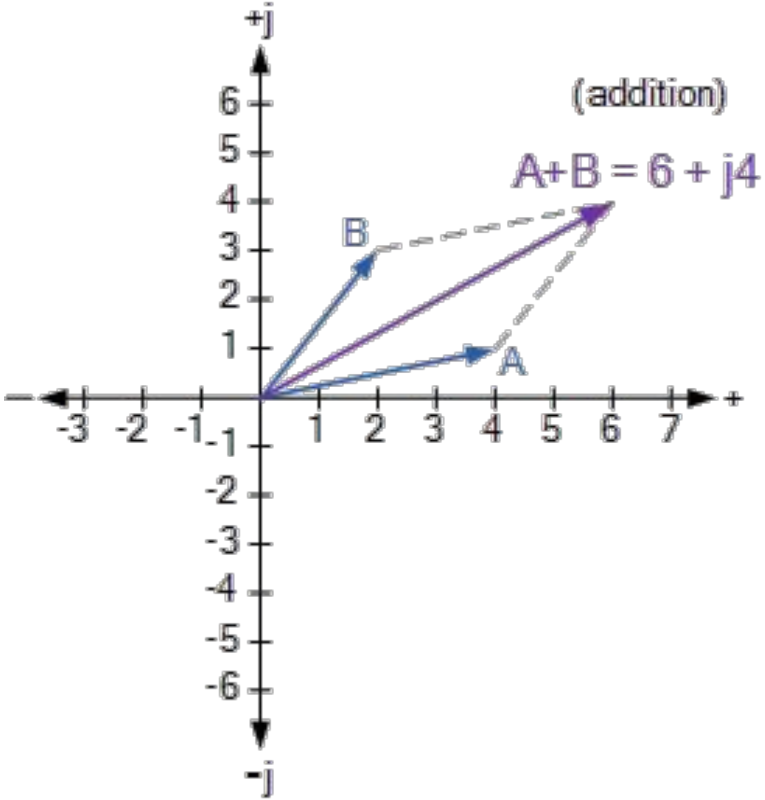
$$A + B = \left(4 + 2\right) + j\left(1 + 3\right) = 6 + j4$$

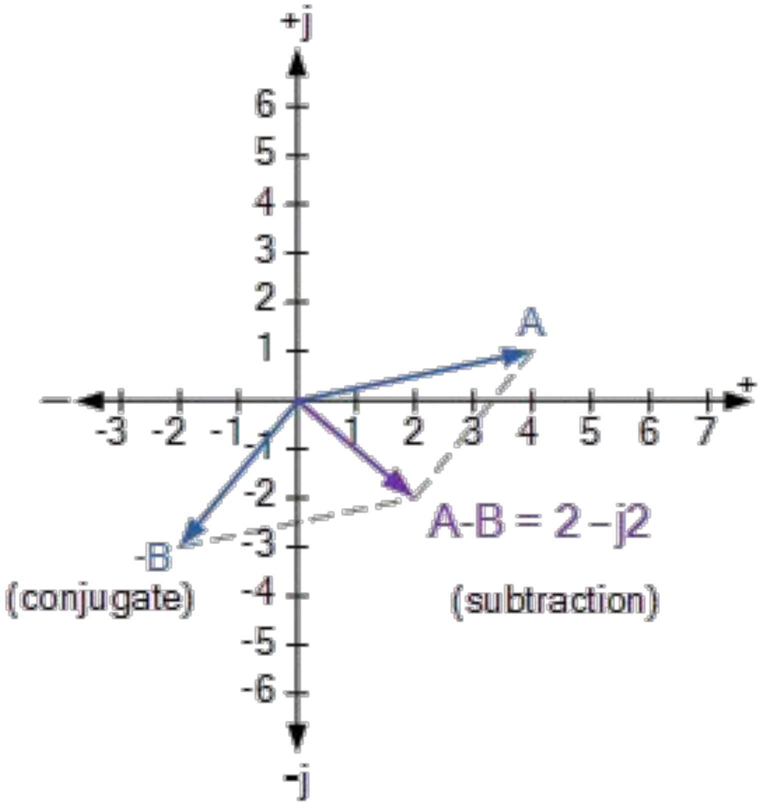
Subtraction

$$A - B = \left(4 + j1\right) - \left(2 + j3\right)$$

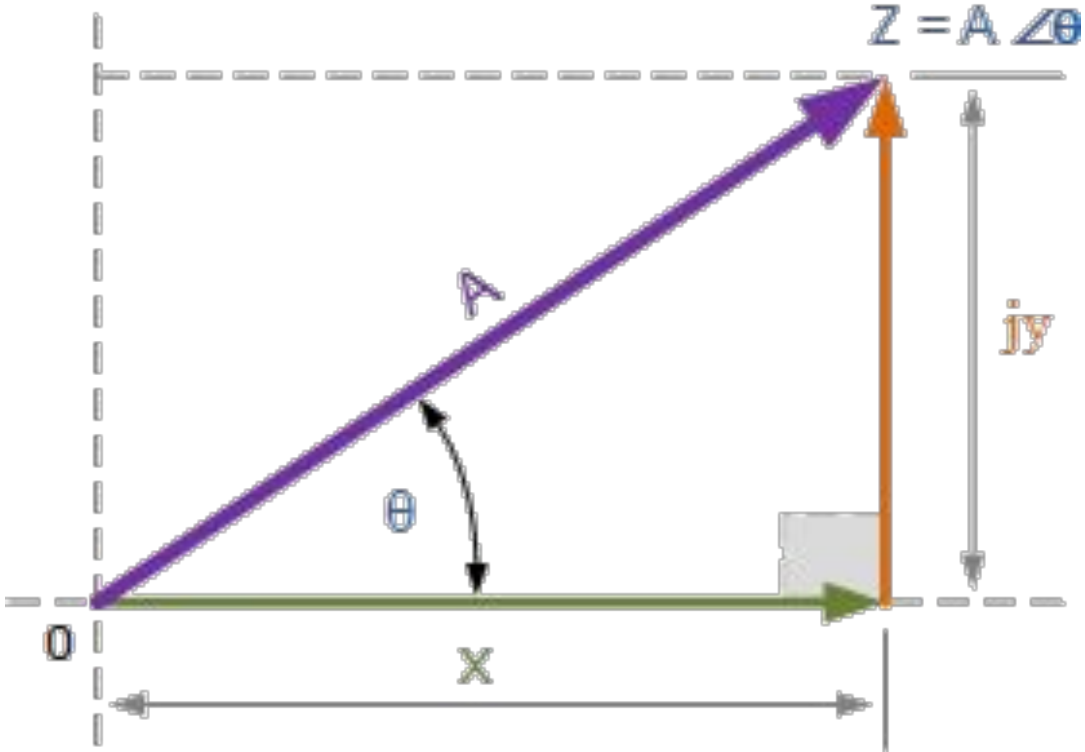
$$A + B = \left(4 - 2\right) + j\left(1 - 3\right) = 2 - j2$$

Graphical Addition and Subtraction





Complex Numbers using Polar Form



Complex Numbers using Polar Form

$$A^2 = x^2 + y^2$$

$$A = \sqrt{x^2 + y^2}$$

$$\text{Also,} \quad x = A.\cos\theta, \quad y = A.\sin\theta$$

$$\theta = \tan^{-1}\frac{y}{x}$$

Converting between Rectangular Form and Polar Form

Converting

Polar Form into Rectangular Form, (P→R)

$$6\angle 30^{o} = x + jy$$

However,

$$x = A.\cos\theta \quad y = A.\sin\theta$$

Therefore,

$$\begin{aligned} 6\angle 30^{0} &= (6\cos\theta) + j(6\sin\theta) \\ &= (6\cos 30^{o}) + j(6\sin 30^{o}) \\ &= (6 \times 0.866) + j(6 \times 0.5) \\ &= 5.2 + j3 \end{aligned}$$

Converting between Rectangular Form and Polar Form

Converting

Rectangular Form into Polar Form, (R→P)

$$(5.2 + j3) = A\angle\theta$$

$$\text{where:} \quad A = \sqrt{5.2^2 + 3^2} = 6$$

$$\text{and} \quad \theta = \tan^{-1}\frac{3}{5.2} = 30^o$$

$$\text{Hence,} \quad (5.2 + j3) = 6\angle30^0$$

Polar Form Multiplication and Division

$$Z_1 \times Z_2 = A_1 \times A_2 \angle \theta_1 + \theta_2$$

Multiplying together 6 ∠30o and 8 ∠– 45o in polar form gives us.

$$Z_1 \times Z_2 = 6 \times 8 \angle 30^\circ + (-45^\circ) = 48 \angle -15^\circ$$

Reference

https://www.electronics-tutorials.ws