## Applied Mathematics - IV

P. Pages: 3

Time : Three Hours
*1170*

AHK/KW/19/2127
Max. Marks : 80

Notes : 1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Assume suitable data whenever necessary.
9. Illustrate your answers whenever necessary with the help of neat sketches.
10. Use of non programmable calculator is permitted.
11. Table for area under standard normal curve is permitted.

1. a) Define:
i) Rise time.
ii) Peak time and.
iii) Settling time.
b) Obtain the mathematical model and transfer function for the system given below. Also state the order of system.

2. a) Discuss time response of a general second order system in a unit step input.
b) A unity feedback system is characterized by an open loop transfer function.
$G(5)=\frac{\mathrm{k}}{\mathrm{s}(\mathrm{s}+10)}$.
Determine the gain k so that the system will have a damping ratio of 0.5 for this value of k , determine the rise time, peak time and setting time for a unit step input.
3. a) If $Z\{f(n)\}=F(z)$ then prove that
$Z\{f(n+k)\}=z^{k}\left[F(z)-\sum_{i=0}^{k+1} f(i) \cdot z^{-i}\right]$, for $k>0$.
b) Find inverse z -transform of
$F(z)=\frac{z^{2}+z}{(z+1)\left(z^{2}+1\right)}$.

## OR

4. a) State convolution theorem of $z$-transform and hence find

$$
z^{-1}\left[\frac{z^{2}}{(z-1) \varepsilon-3)}\right] .
$$

b) Solve using z -transform $\mathrm{y}_{\mathrm{n}+2}+\mathrm{y}_{\mathrm{n}}=2$, given $\mathrm{y}_{0}=\mathrm{y}_{1}=0$.
5. a) Define :
i) Fuzzy set,
ii) $\quad \alpha$-level set and
iii) Normalized fuzzy set.
b) If the universe of discourse is $\mathrm{x}=\{0,1,2,3,4\} 5$ then write the fuzzy set A whose membership grade function is
$\mu_{A}(x)=\frac{x}{x+2}$.
Also find $\overline{\mathrm{A}}$, scalar cardinality of A and 0.2 cut of A.

## OR

6. a) Find A B, A B and A x B, where
$A=\frac{0.9}{1}+\frac{0.7}{3}+\frac{0.2}{4}+\frac{0.3}{6}$ and
$B=\frac{0.1}{2}+\frac{0.4}{3}+\frac{0.5}{4}+\frac{0.8}{5}$ are defined on the universe
$\mathrm{U}=\{1,2,3,4,5,6\}$.
b) For the fuzzy sets A \& B, define
$\mathrm{A}-\mathrm{B}$ and $\mathrm{A} \oplus \mathrm{B}$.
If $A=\frac{0.2}{x_{1}}+\frac{0.5}{x_{2}}+\frac{0.6}{x_{3}}$ and
$\mathrm{B}=\frac{0.1}{\mathrm{x}_{1}}+\frac{0.4}{\mathrm{x}_{2}}+\frac{0.5}{\mathrm{x}_{3}}$, then find
$A-B$ and $A \oplus B$.
7. a) Find by newton Raphson method, the root of the equation $\cos x=x e^{x}$.
b) Solve by Gauss-Seidel method :

$$
\begin{aligned}
& 2 x+10 y+z=13 \\
& 2 x+2 y+10 z=14 \\
& 10 x+y+z=12
\end{aligned}
$$

## OR

8. a) Find a real root of the equation $x^{4}-11 x+8=0$ correct to four decimal places by method of false position.
b) Solve by Crout's method, the system of equations.

$$
\begin{aligned}
& x+y+z=1 \\
& 3 x+y-3 z=5 \\
& x-2 y-5 z=10
\end{aligned}
$$

9. a) Using modified Euler's method, solve the equation :
$\frac{d y}{d x}=x^{2}+y$ for $x=0.1$, given that $y(0)=1$ and $h=0.05$.
b) Solve by Runge-Kutta method :
$\frac{d y}{d x}=y z+x, \frac{d z}{d x}=x z+y$ for $x=0.2$, given that $y(0)=1, z(0)=1$ and $h=0.2$.

## OR

10. a) Find the series solution of the differential equation using Taylor's series method.
$\frac{d y}{d x}=\frac{1}{2}\left(y^{2}+x y^{2}\right), y(0)=1$.
and hence find $y(0.1)$ correct upto $3{ }^{\text {rd }}$ decimal place.
b) Solve the following differential equation by Milne's predictor corrector method.
$\frac{d y}{d x}=1+\mathrm{xy}^{2}, \mathrm{y}(0)=1, \mathrm{y}(0.1)=1.105, \mathrm{y}(0.2)=1.223, \mathrm{y}(0.3)=1.355$.
Find $y(0.4)$ and $y(0.5)$.
11. a) Three machines $A, B, C$ produce respectively $60 \%, 30 \%$ and $10 \%$ of the total number of items of a factory. The percentages of defective output of these machines are respectively $2 \%, 3 \%$ and $4 \%$. An item is selected at random and is found defective. Find the probability that the item was produced by machine C .
b) Find moment generating function and first four moments about origin for the r.v. X
having density function
$f(x)=\left\{\begin{array}{ll}\alpha e^{-\alpha x}, & x>0 \\ 0, & x \leq 0\end{array}\right.$.
c) If $10 \%$ of bolts produced by a machine are defective, determine the probability that out of 100 bolts chosen at random.
i) One
ii) None
iii) At Most 2 bolts will be defective.

## OR

12. a) Find the mean, median and mode for a random variable having probability density function.

$$
\begin{aligned}
\mathrm{f}(\mathrm{x}) & =4 \mathrm{x}\left(1-\mathrm{x}^{2}\right), & & 0<\mathrm{x}<1 \\
& =0, & & \text { otherwise. }
\end{aligned}
$$

b) A random variable X has density function
$\mathrm{f}(\mathrm{x})= \begin{cases}\mathrm{kx} \mathrm{x}^{2}, & 1 \leq \mathrm{x} \leq 2 \\ \mathrm{kx}, & 2<\mathrm{x}<3 \\ 0, & \text { otherwise }\end{cases}$
find the constant k and the distribution function.
c) Find mathematical expectation of discrete random variable x whose probability function is

$$
f(x)=(1 / 2)^{x}, x=1,2,3,-\cdots-
$$

