Mesh Analysis

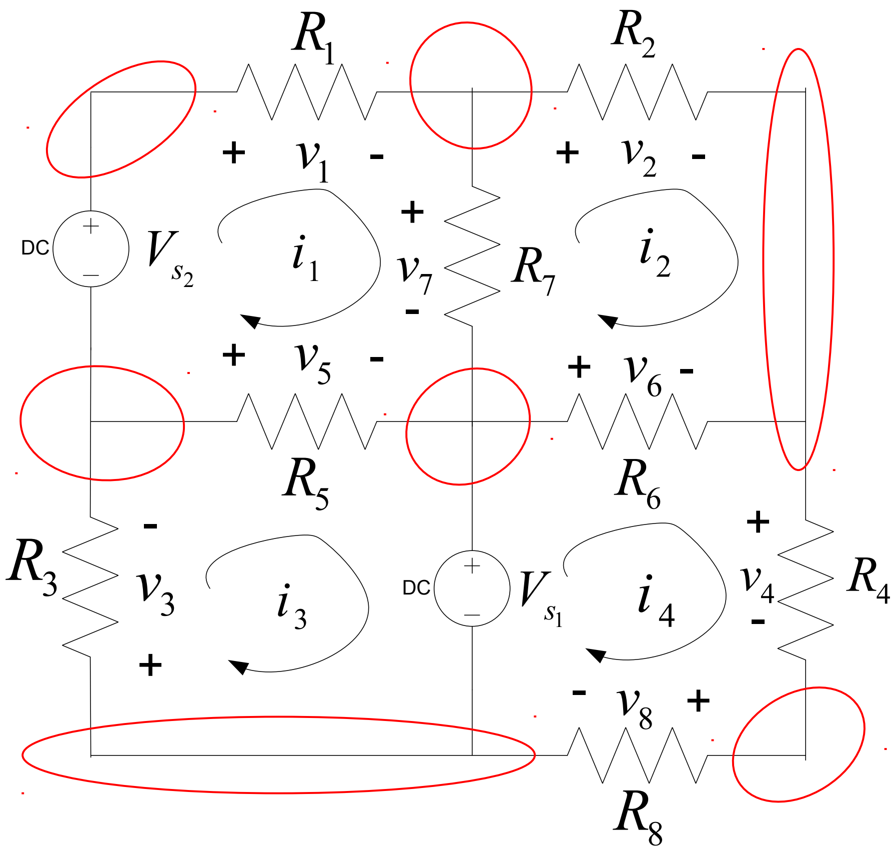
Mesh Analysis

- Mesh analysis applies KVL to find unknown currents.
- A mesh is a loop that does not contain any other loops.
- The current through a mesh is known as the mesh current.
- Assume for simplicity that the circuit contains only voltage sources.

Mesh Analysis Steps

1. Assume mesh currents i_1, i_2, i_3, … i_l, to the l meshes.
2. Apply KVL to each of the l meshes and use Ohm’s law to express the voltages in terms of the mesh currents.
3. Solve the l resulting simultaneous equations to find the mesh currents or form a matrix equation using mesh equations.
4. Find the individual parameters by solving Mesh equations.

Example

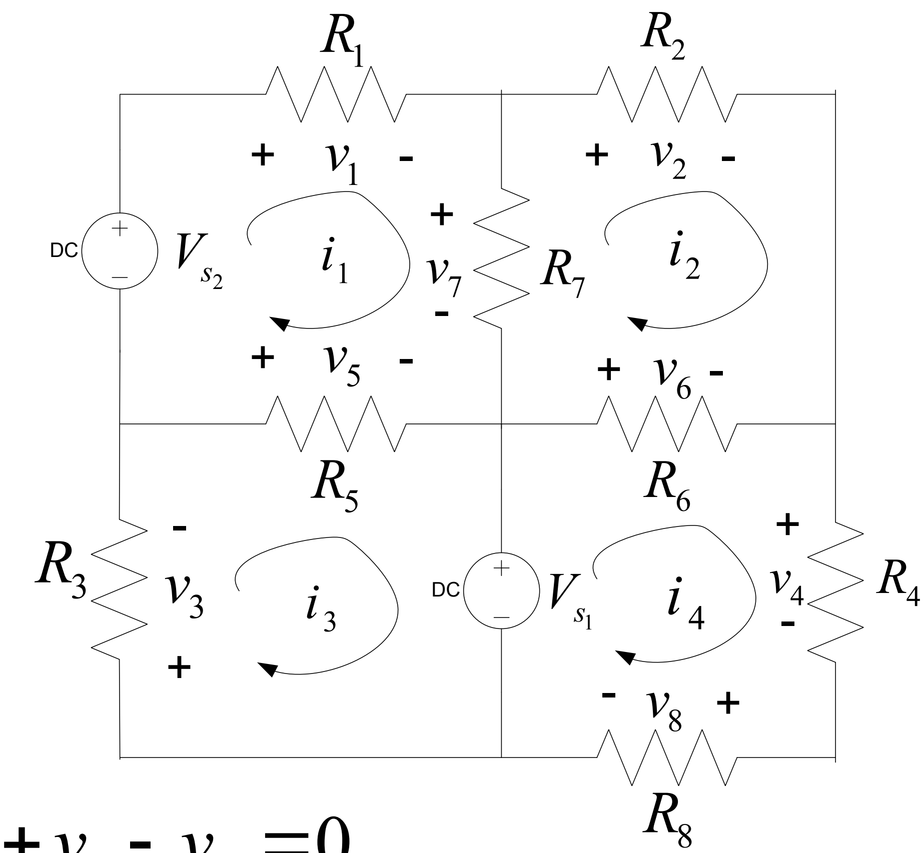


Number of nodes, $n =$ **7**

Number of loops, $l =$ **4**

Number of branches, $b =$ **10**

Example



Apply KVL to each mesh

Mesh 1: $-V_{s_2} + v_1 + v_7 - v_5 = 0$

Mesh 2: $v_2 - v_6 - v_7 = 0$

Mesh 3: $v_5 + v_{s_1} + v_3 = 0$

Mesh 4: $v_4 + v_8 - V_{s_1} + v_6 = 0$

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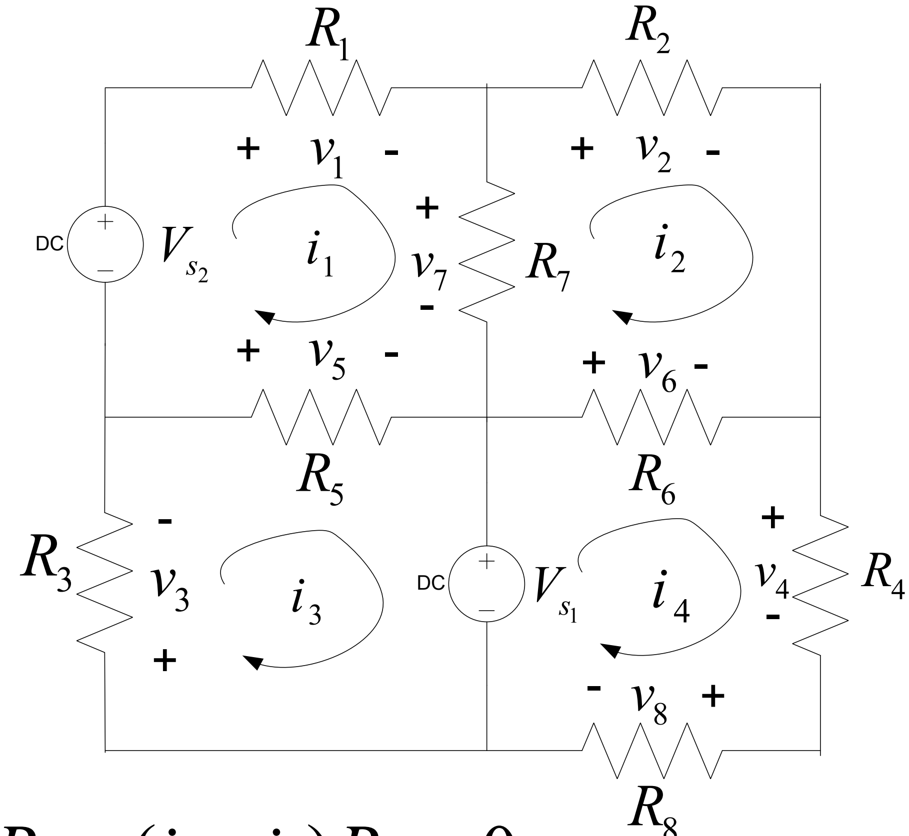


Express the voltage in terms of the mesh currents:

Mesh 1: $-V_{s_2} + i_1 R_1 + (i_1 - i_2)R_7 + (i_1 - i_3)R_5 = 0$

Mesh 2: $i_2 R_2 + (i_2 - i_4)R_6 + (i_2 - i_1)R_7 = 0$

Mesh 3: $(i_3 - i_1)R_5 + V_{s_1} + i_3 R_3 = 0$

Mesh 4: $i_4 R_4 + i_4 R_8 - V_{s_1} + (i_4 - i_2)R_6 = 0$

Mesh 1: $-V_{s_2} + i_1 R_1 + (i_1 - i_2)R_7 + (i_1 - i_3)R_5 = 0$

Mesh 2: $i_2 R_2 + (i_2 - i_4)R_6 + (i_2 - i_1)R_7 = 0$

Mesh 3: $(i_3 - i_1)R_5 + V_{s_1} + i_3 R_3 = 0$

Mesh 4: $i_4 R_4 + i_4 R_8 - V_{s_1} + (i_4 - i_2)R_6 = 0$

Mesh 1: $(R_1 + R_5 + R_7)i_1 - R_7 i_2 - R_5 i_3 = V_{s_2}$

Mesh 2: $-R_7 i_1 + (R_2 + R_6 + R_7)i_2 - R_6 i_4 = 0$

Mesh 3: $-R_5 i_1 + (R_3 + R_5)i_3 = -V_{s_1}$

Mesh 4: $-R_6 i_2 + (R_4 + R_6 + R_8)i_4 = V_{s_1}$

Mesh 1: $(R_1 + R_5 + R_7)i_1 - R_7 i_2 - R_5 i_3 = V_{s_2}$

Mesh 2: $- R_7 i_1 + (R_2 + R_6 + R_7)i_2 - R_6 i_4 = 0$

Mesh 3: $- R_5 i_1 + (R_3 + R_5)i_3 = - V_{s_1}$

Mesh 4: $- R_6 i_2 + (R_4 + R_6 + R_8)i_4 = V_{s_1}$

$$\begin{pmatrix} R_1 + R_5 + R_7 & - R_7 & - R_5 & 0 \\ - R_7 & R_2 + R_6 + R_7 & 0 & - R_6 \\ - R_5 & 0 & R_3 + R_5 & 0 \\ 0 & - R_6 & 0 & R_4 + R_6 + R_8 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{pmatrix} = \begin{pmatrix} V_{s_2} \\ 0 \\ - V_{s_1} \\ V_{s_1} \end{pmatrix}$$

$$
\begin{pmatrix} R_1 + R_5 + R_7 & -R_7 & -R_5 & 0 \\ -R_7 & R_2 + R_6 + R_7 & 0 & -R_6 \\ -R_5 & 0 & R_3 + R_5 & 0 \\ 0 & -R_6 & 0 & R_4 + R_6 + R_8 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{pmatrix} = \begin{pmatrix} V_{s_2} \\ 0 \\ -V_{s_1} \\ V_{s_1} \end{pmatrix}
$$

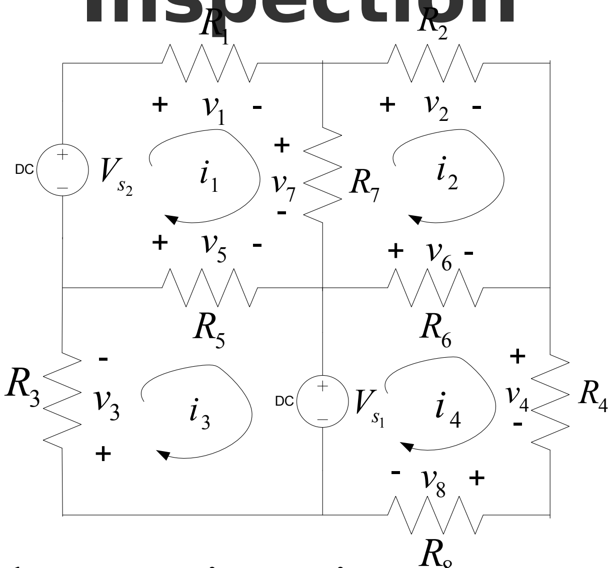
$$
\mathbf{Ri} = \mathbf{v}
$$

$\mathbf{R}$ is an $l \times l$ *symmetric* resistance matrix

$\mathbf{i}$ is a $1 \times l$ vector of mesh currents

$\mathbf{v}$ is a vector of voltages representing “known” voltages

Writing the Mesh Equations by Inspection



$$\begin{pmatrix} R_1+R_5+R_7 & -R_7 & -R_5 & 0 \\ -R_7 & R_2+R_6+R_7 & 0 & -R_6 \\ -R_5 & 0 & R_3+R_5 & 0 \\ 0 & -R_6 & 0 & R_4+R_6+R_8 \end{pmatrix}\begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{pmatrix} = \begin{pmatrix} V_{s_2} \\ 0 \\ -V_{s_1} \\ V_{s_1} \end{pmatrix}$$

•The matrix **R** is symmetric, $r_{kj} = r_{jk}$ and all of the off-diagonal terms are negative or zero.

The r_{kk} terms are the sum of all resistances in mesh *k*.

The r_{kj} terms are the negative sum of the resistances common to BOTH mesh *k* and mesh *j*.

The v_k (the k^{th} component of the vector **v**) = the algebraic sum of the independent voltages in mesh *k*, with voltage rises taken as positive.

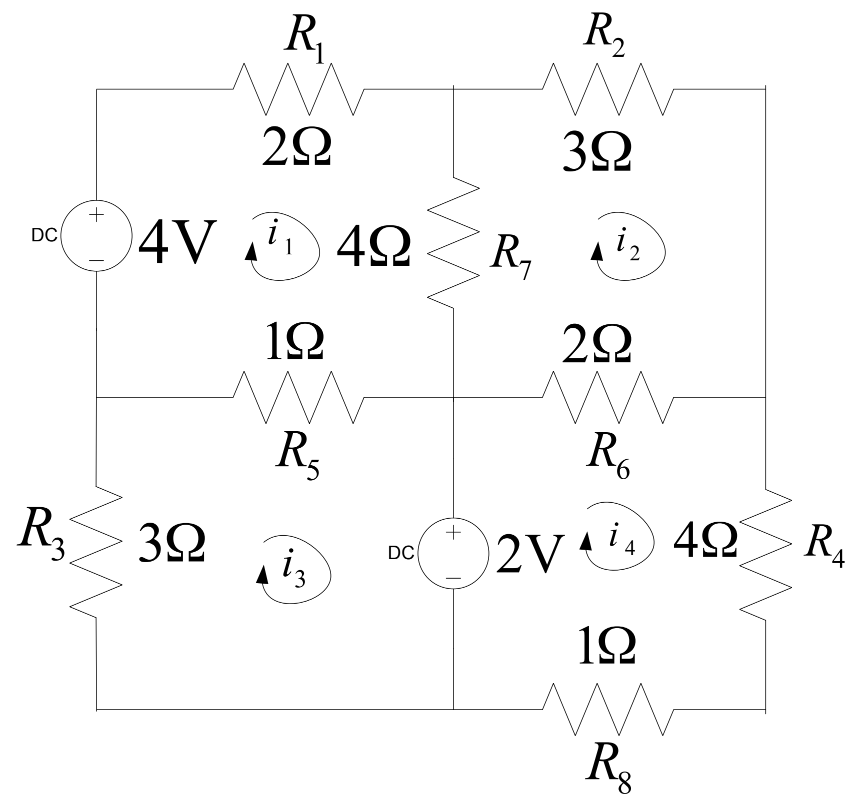
MATLAB Solution of Mesh Equations

$$\begin{pmatrix} R_1+R_5+R_7 & -R_7 & -R_5 & 0 \\ -R_7 & R_2+R_6+R_7 & 0 & -R_6 \\ -R_5 & 0 & R_3+R_5 & 0 \\ 0 & -R_6 & 0 & R_4+R_6+R_8 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{pmatrix} = \begin{pmatrix} V_{s_2} \\ 0 \\ -V_{s_1} \\ V_{s_1} \end{pmatrix}$$

$$\mathbf{Ri} = \mathbf{v}$$

$$\mathbf{i} = \mathbf{R}^{-1}\mathbf{v}$$

Test with numbers



$$\begin{pmatrix} 2+4+1 & -4 & -1 & 0 \\ -4 & 3+2+4 & 0 & -2 \\ -1 & 0 & 3+1 & 0 \\ 0 & -2 & 0 & 2+4+1 \end{pmatrix}\begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ -2 \\ 2 \end{pmatrix}$$

Test with numbers

$$\begin{pmatrix} 2+4+1 & -4 & -1 & 0 \\ -4 & 3+2+4 & 0 & -2 \\ -1 & 0 & 3+1 & 0 \\ 0 & -2 & 0 & 2+4+1 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ -2 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 7 & -4 & -1 & 0 \\ -4 & 9 & 0 & -2 \\ -1 & 0 & 4 & 0 \\ 0 & -2 & 0 & 7 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ -2 \\ 2 \end{pmatrix}$$

$$\mathbf{Ri} = \mathbf{v}$$