

Applied Mathematics-IV

P. Pages : 4

Time : Three Hours

0637

NRT/KS/19/3363

Max. Marks : 80

- Notes :
1. All questions carry marks as indicated.
 2. Solve Question 1 OR Questions No. 2.
 3. Solve Question 3 OR Questions No. 4.
 4. Solve Question 5 OR Questions No. 6.
 5. Solve Question 7 OR Questions No. 8.
 6. Solve Question 9 OR Questions No. 10.
 7. Solve Question 11 OR Questions No. 12.
 8. Due credit will be given to neatness and adequate dimensions.
 9. Assume suitable data whenever necessary.
 10. Illustrate your answers whenever necessary with the help of neat sketches.
 11. Use of non programmable calculator is permitted.
 12. Use of normal distribution table is permitted.

- 1.** a) Define transfer function of the system and obtain transfer function of series R -C circuit. **6**
- b) Obtain unit step response of unity feedback system whose open loop transfer function is **6**
- $$G(s) = \frac{4}{s(s+5)}$$

OR

- 2.** a) Give the block diagram of a simple closed loop control system and derive expression for its transfer function. **6**
- b) Define: **6**
- i) Step signal ii) Ramp signal iii) Parabolic signal
and also find their Laplace transform.
- 3.** a) Find the Z-transform of $\cos n\theta$ and hence find $Z\{a^n \cos n\theta\}$. **6**
- b) If $Z\{f(n)\} = F(z)$ then prove that $Z\{f(n+k)\} = z^k \left[F(z) - \sum_{i=0}^{k-1} f(i) \cdot z^{-i} \right]$, $k > 0$. **6**

OR

- 4.** a) By using convolution theorem. **6**
- find $Z^{-1} \left\{ \frac{z^2}{(z-1)(z-3)} \right\}$

- b) Solve the difference equation **6**
 $y_{n+2} + 4y_{n+1} + 3y_n = 2^n, y_0 = 0, y_1 = 1$
 using Z-transform.

- 5.** a) Define: **6**
 i) Fuzzy set, ii) α - level set and iii) Normalized fuzzy set.

- b) Find $A \cap B, A \cup B$ and $A \oplus B$, **6**
 where

$$A = \frac{0.9}{1} + \frac{0.7}{3} + \frac{0.2}{4} + \frac{0.3}{6}$$

and

$$B = \frac{0.1}{2} + \frac{0.4}{3} + \frac{0.5}{4} + \frac{0.8}{5}$$

are defined on $U = \{1, 2, 3, 4, 5, 6\}$.

OR

- 6.** a) Define measure of fuzziness of a fuzzy set. Find the measure of fuzziness for the fuzzy **6**
 set $A = \frac{0.2}{x_1} + \frac{0.4}{x_2}$.

- b) If the universe of discourse is $X = \{0, 1, 2, 3, 4, 5\}$, then write the fuzzy set A whose **6**
 membership grade function is $\mu_A(x) = \frac{x}{x+2}$. Also find \bar{A} , scalar cardinality of A
 and 0.2 cut of A.

- 7.** a) Find by Newton - Raphson method, the real root of the equation $3x - \cos x - 1 = 0$. **5**

- b) Apply Crout's method to solve the equations. **7**
 $3x + 2y + 7z = 4,$
 $2x + 3y + z = 5,$
 $3x + 4y + z = 7.$

OR

- 8.** a) Find a real root of the equation **6**
 $x \log_{10} x = 1.2$
 by Regula - Falsi method
 Correct to four decimal places.

- b) Solve **6**
 $20x + y - 2z = 17$
 $3x + 20y - z = 18$
 $2x - 3y + 20z = 25$
 by Gauss - Seidel method.

9. a) Using modified Euler's method, solve the equation: 7
- $$\frac{dy}{dx} = x + \sqrt{y}, y(0) = 1$$
- for the range $0 \leq x \leq 0.4$,
with $h = 0.2$.

- b) Compute $y(0.2)$, given 7
- $$\frac{dy}{dx} + y + x y^2 = 0, y(0) = 1$$
- by taking $h = 0.1$ using Runge - Kutta method of fourth order (correct to four decimals).

OR

10. a) Employ Taylor's method to obtain approximate value of y at $x = 0.2$ for the differential equation 7
- $$\frac{dy}{dx} = 2y + 3e^x, y(0) = 0.$$
- compare the numerical solution obtained with the exact solution.

- b) Solve the following differential equation by Milne's predictor corrector method: 7
- $$\frac{dy}{dx} = \frac{1}{2} (1 + x^2) y^2, y(0) = 1,$$
- $y(0.1) = 1.06, y(0.2) = 1.12, y(0.3) = 1.21.$
Evaluate $y(0.4)$ and $y(0.5)$.

11. a) Three machines A, B and C produce respectively 50%, 30% and 20% of the items in a factory. The percentage of defective output of these machines are 3%, 4% and 5% respectively. One item is selected at random and is found to be defective. Find the probability that the item was produced by machine A. 6

- b) Let $f(x) = \frac{C}{3^x}, x = 1, 2, 3, \dots$ is the probability function of a random variable X. 5
- Find
- i) Constant C and ii) $P(X \geq 3)$.

- c) Let X be a random variable with density function 7
- $$f(x) = \begin{cases} 2e^{-2x}, & x \geq 0 \\ 0, & \text{otherwise.} \end{cases}$$
- Find
- i) $E(X)$ ii) $E(X^2 + 5)$
iii) $\text{Var}(X)$ iv) S. D. of X.

OR

12. a) Find the moment generating function of random variable. **5**

$$X = \begin{cases} 1, & \text{Prob. } \frac{1}{2} \\ -1, & \text{Prob. } \frac{1}{2} \end{cases}$$

Hence find first four moments about origin.

b) Find the coefficient of **7**

- i) Skewness and
- ii) Kurtosis of distribution:

$$f(x) = \begin{cases} \frac{4x(9-x^2)}{81}, & 0 \leq x \leq 3 \\ 0, & \text{otherwise.} \end{cases}$$

c) The number of monthly breakdowns of a computer, is a random variable having a Poisson distribution with mean equal to 1.8. Find the probability that this computer will function for a month **6**

- i) Without breakdown and
- i) With at least one breakdown.
