B.E. (Electronics Engineering / Electrical Engineering (Electronics & Power) / Electronics Telecommunication Engineering / Electronics Communication Engineering / Mechanical Engineering) Third Semester (C.B.S.)

# Applied Mathematics - III

P. Pages : 4 **NRT/KS/19/3297/3302/3307/33**

Time : Three Hours *0184* Max. Marks : 80

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Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Use of non programmable calculator is permitted.

**1.** a) If $L\{f(t)\} = \bar{f}(s)$ then Prove that $L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty \bar{f}(s)\, ds.$ **6**

Hence find the Laplace transform of $\frac{\cos at - \cos bt}{t}$

b) Express $f(t) = \begin{cases} t^2 & 0 < t < 2 \\ 4t & t > 2 \end{cases}$ **6**

in term of unit step function and find its Laplace transform

**OR**

**2.** a) Use convolution theorem to find **6**

$$L^{-1}\left\{\frac{s^2}{(s^2 + a^2)(s^2 + b^2)}\right\}$$

b) Solve the differential equation by Laplace Transform **6**

$$\frac{dy}{dx} + 2y + \int_0^t y\, dt = \sin t \text{ given } y(0) = 1.$$

**3.** a) Find the Fourier series for the function **6**

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi} & -\pi < x < 0 \\ 1 - \frac{2x}{\pi} & 0 < x < \pi \end{cases}$$

and hence show that $f(x) = \frac{4}{\pi}\left\{\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + - - \right\} -$

b) Find Fourier transform of $e^{-|x|}$ and hence show that **6**

$$\int_0^\infty \frac{x \sin mx}{1+x^2} dx = \frac{\pi}{2} e^{-m} \quad m > 0$$

**OR**

**4.** a) Find half range Fourier sine series for the function $f(x) = x^3$ in the interval $0 < x < 1$. **6**

b) Solve the integral equation $\int_0^\infty f(x) \cos \alpha x \, dx = \begin{cases} 1-\alpha & 0 \le \alpha \le 1 \\ 0 & \alpha > 1 \end{cases}$ **6**

and hence evaluate $\int_0^\infty \frac{\sin^2 t}{t} dt$

**5.** Find the extremals of $v(y(x)) = \int_a^b \frac{1+(y')^2}{(y')^2} dx$ **6**

**OR**

**6.** Find the extremal of the functional **6**

$$v(y(x)) = \int_{x_1}^{x_2} \left\{ x^2 (y')^2 + 2y^2 + 2xy \right\} dx$$

**7.** a) If f(z) is analytic function of z, then prove that **6**

$$\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^2 = 4 |f'(z)|^2$$

b) Evaluate using Cauchy integral formula $\int_C \frac{\cos \pi z^2}{(z-1)(z-2)} dz$ when C is the circle **6**

i) $|z| = 3$ ii) $|z+i| = 1.5$

c) Expand the function using Laurentz series $f(z) = \frac{z^2 - 4}{(z+1)(z+4)}$ valid for **6**

i) $|z| < 1$
ii) $1 < |z| < 4$
iii) $|z| > 4$

**OR**

**8** a) Evaluate by Cauchy Residue theorem $\int_C \frac{12z - 7}{(z-1)^2 (2z+3)} dz$ where C is the circle **6**

i) $|z| = 2$ ii) $|z + i| = \sqrt{3}$

b) If $u = e^{x}[x\cos y - y\sin y]$ show that u is harmonic function. Find $v$ such that $f(z) = u + iv$ is an analytic function. **6**

c) Evaluate $\int_{0}^{2\pi} \frac{\cos 3\theta}{5 - 4\cos\theta}\, d\theta$ by contour integration. **6**

**9.** a) Solve $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$. **7**

b) Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y\cos x$ **7**

**OR**

**10.** a) Solve the equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial y} + 2u$ Given that $u = 0$, $\frac{\partial u}{\partial x} = 1 + e^{-3y}$, when x = 0 for all values of y using method of separation of variable. **7**

b) Using Laplace transform method solve **7**

$$\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial x^2} - 4y$$

$$y(0, t) = 0 = y(\pi, t)$$

$$y(x, 0) = 0\ 6\sin x - 4\sin 2x$$

**11.** a) Investigate the linear dependence of the rector $x_1 = (1, 2, 4)$, $x_2 = (2, -1, 3)$ $x_3 = (0, 1, 2)$ $x_4 = (-3, 7, 2)$ **6**

b) Find eigan value, eigan vector and modal matrix of **6**

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

c) Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ **6**

and hence find the matrix represented by

i) $A^4 - 5A^3 + 8A^2 - 2A + I$

ii) $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$

**OR**

**12.** a) Use Sylvester's theorem to show that **6**

$$e^{A} = e^{x}\begin{bmatrix} \cos hx & \sin hx \\ \sin hx & \cos hx \end{bmatrix}$$

where $A = \begin{bmatrix} x & x \\ x & x \end{bmatrix}$

b) Solve the differential equation by matrix method $\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 6y = 0$ given **6**

$y(0) = 2 \; y'(0) = 5$

c) Reduce the quadratic form $8x_1^2 + 7x_2^2 + 3x_3^2 - 12x_1x_2 + 4x_1x_3 - 8x_2x_3$ to canonical form by an orthogonal transformation. **6**

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